B.Sc.–I 1st Semester (Hons.) Examination,

November–2014

MATHS

Paper–BHM–113

Solid Geometry

Time allowed : 3 hours] *[Maximum marks : 60*

Note : *Attempt five questions, selecting **one** question from each section. **Question No. 9 is compulsory.***

Section–I

1. (a) Trace the conic

$$8x^2 - 4xy + 5y^2 - 16x - 14y + 17 = 0$$

(b) To find the pole of the line $\ell x + my + n = 0$ with respect to the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

2. (a) Show that the points of contact of the tangents of the confocal conics $\dfrac{x^2}{a^2+\lambda} + \dfrac{y^2}{b^2+\lambda} = 1$ which are also tangents to the parabola $y^2 = 4x\,(a^2 - b^2)^{1/2}$, be on a straight line.

(b) In any conic, prove that the sum of reciprocal of two perpendicular focal chords is constant.

Section–II

3. (a) Find the equation of the sphere, through the circle $x^2 + y^2 + z^2 = 1, 2x + 4y + 5z = 6$ and touching the plane $z = 0$.

   (b) Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point $(1, -2, 1)$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$

4. (a) Prove that the equation $2y^2 - 8yz - 4zx - 8xy + 6x - 4y - 2z + 5 = 0$ represents a cone, whose vertex is $\left(-\frac{7}{6}, \frac{1}{3}, \frac{5}{6}\right)$

   (b) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 1 = 0$ having its generators parallel to the line $x = y = z$.

Section–III

5. (a) Find the point of contact at which the plane $\ell x + my + hz = p$ touches the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

   (b) To prove that the six normals from a point to an ellipsoid lie on a curve of second degree.

6. (a) Find the equations of the polar of the line $\frac{x-1}{5}=\frac{y-3}{7}=\frac{z+5}{2}$ w.r.t. the conicoid $x^2+3y^2-7z^2-21=0$ in symmetrical form.

(b) To find the equation of the enveloping cylinder of the conicoid $ax^2+by^2+cz^2=1$ whose generators are parallel to the line $\frac{x}{\ell}=\frac{y}{m}=\frac{z}{n}$.

Section–IV

7. (a) Prove that the normals from (α, β, γ) to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lie on the cone

$$\frac{\alpha}{x-\alpha}-\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0.$$

(b) Reduce the following equation to the standard form

$$2x^2+5y^2+2z^2-2yz+4zx-2xy+14x-16y+14z+26=0$$

8. (a) Find the equations of the generating lines of the hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$, which pass through the point $(2, 3, -4)$ and $\left(2, -1, \frac{4}{3}\right)$.

(b) Show that the two confocal paraboloids cut everywhere at right angles.

Section–V

9. (a) Define pole and polar of a conic.

(b) Find the polar equation of a circle with centre (x_1, θ_1) and radius a.

(c) Find the condition that the plane $\ell x + my + nz = p$ may touch the sphere $x^2 + y^2 + z^2 = a^2$.

(d) Define right circular cone, its vertex and axis.

(e) Show that the plane $8x - 6y - z = 5$ touches the paraboloid $\frac{x^2}{2} - \frac{y^2}{3} = z$.

(f) Find the centre of the conicoid

$$3x^2 + 6yz - y^2 - z^2 - 6x + 6y - 2z - 2 = 0$$